ICME 2023

Proceedings of ICME 2023
14th International Conference on Mechanical Engineering
18-19 December 2023, BUET, Dhaka, Bangladesh

SSRN tomorrow's research today

Effects of Variable Permeability and Viscous Dissipation on an Unsteady MHD Free Convective Transport over a Vertical Permeable Sheet

Tanzila Akter Nabila[1], Md. Hasanuzzaman[1,a)], Md. Mosharof Hossain[2]

[1]*Department of Mathematics, Khulna University of Engineering and Technology, Khulna-9203, Bangladesh*
[2]*Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh*

[a)]Corresponding author: hasanuzzaman@math.kuet.ac.bd

Abstract- The roles of the variable permeability and Eckert number on the time-dependent MHD natural convective transport over a vertical perforated plate are analyzed in this study. The model that formed highly nonlinear governing equations is changed using similarity substitutions. Then the coupled ODEs are solved by inserting the finite difference method (FDM). The numerical solutions of the fluid temperature, velocity, and concentration are presented graphically. The tabular form explored here is the local skin friction coefficient ($f'(0)$) the heat transfer rate $(-\theta'(0))$ and the mass transfer rate $(-\phi'(0))$. The results give the temperature and motion of the fluid improvement for growing values of the Eckert number. Also, the fluid velocity enhances for growing values of the Darcy number. The values of $f'(0)$ increase by about 10% but the heat transfer rate lessens by about 43% due to rising values of the Eckert number (0.5- 2.5). The obtained numerical outcomes are compared with the previously published study. The comparison is given to be an excellent contract.

INTRODUCTION

The magneto-hydrodynamics (MHD) free convective flow has attracted many researchers due to its applications in several engineering problems in plasma studies, boundary layer flow control, geothermal energy extraction nuclear reactors, and MHD pumps and generators. A lot of researchers have studied in this area. Primarily, Pavlov [1] analyzed the hydromagnetic fluid flow attributed to the deformity of the plane surface. The study of heat production or absorption effect on heat passage is significant because these influences are conclusive in heat transference. Many researchers examined the effect of heat-absorbing or generating fluids for various flow regimes. Taiwo and Dauda [2] investigated the time-dependent flow of magnetized heat absorbing or generating viscous fluid. For all fluid properties, viscosity needs to be considered the most in the research of fluid flow. Prandtl [3] analyzed a field of fluid dynamics by taking into account viscosity and in this way combining theoretical hydraulics and hydrodynamics in the 20th century. The effect of such heat dissipation terms on a time-dependent state is often neglected. The role of the viscous dissipation function may not be ignored from a practical side as it is important in various flow problems. The predominance of heat dissipation on the temperature field is relatively small for much lower velocity methods. A dynamic temperature-related approach cannot ignore the impact of viscous dissipation which is comparable to the temperature difference. The theory of boundary layer is used to explore the heat dissipation influence for both compressible and incompressible flows. Fonsho [4] investigated the influences of the dissipation function on the time-dependent MHD radiating gas flow past a vertical plate. The role of fixed transpiration on an unsteady natural convection elastics' viscous fluid flow upon a permeable plate was explored by Sen [5]. Uwanta et al. [6] analyzed the impact of heat dissipation on viscoelastic fluid flow over an infinite sheet. The influences of heat dissipation on the mixed convective transport of heat-producing or absorbing fluid with the wall conduction were theoretically studied by Ajibade et al. [7]. Very recently, Hasanuzzaman et al. [8] explored the effect of Dufour and Soret on a time-dependent MHD free convective fluid flow upon a vertical permeable sheet. They considered the uniform porous plate in their simulation. Further, the roles of Eckert number and thermal radiation on time-dependent MHD conductive transport across a vertical porous plate were examined by Hasanuzzaman et al. [9].

This article's key objective is to explain the roles of variable permeability and viscous dissipation on time-varying MHD convective heat and mass transport past a vertical perforated sheet. The key novelty of this current study is also extended by Hasanuzzaman et al. [8] by assuming the variable permeability and viscous dissipation under the FDM which has not been investigated yet. Another creativity of this paper is to investigate the present outcomes with those of a previously reported study. Using FDM and the shooting approach in the MATLAB scheme, the numerical result for the temperature, concentration, and velocity equations is visually obtained. Besides, the local skin friction coefficient and the heat and mass transfer rates are found in the tabular representations.

MODEL AND GOVERNING EQUATIONS

Let us consider a time-dependent 2D flow of an electrically conducting and viscous incompressible fluid, along an infinite vertical permeable flat sheet immersed in a permeable medium. The infinite vertical sheet is taken on the x-axis. Also, the vertical sheet is perpendicular to the y-axis. A magnetic field of uniform strength $\mathbf{B} = (0, \mathrm{B}_0)$ is applied transversely to the direction of the flow. U_0 is a velocity where the sheet begins to move impulsively in its own plane. The concentration and temperature are promoted to C_w and T_w. Since the plate is infinite, then $\frac{\partial \mathrm{u}}{\partial \mathrm{x}} \rightarrow 0$ as $x \rightarrow \infty$. For this reason, the fluid velocity is a function of y and t is the function of the physical variables only. The physical model and coordinate systems are plotted in Figure 1.

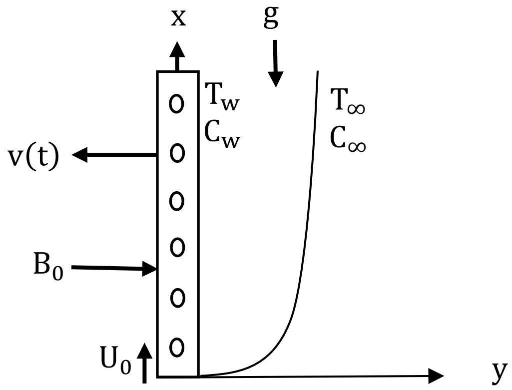

FIGURE 1. Schematic representation of the physical model and coordinates system

Applying the Boussinesq approximation, the e following governing equations Hasanuzzaman et al. [8] are given by

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\nu}{K}u - \frac{\sigma' B_0^2}{\rho}u \tag{2}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} - \frac{\upsilon}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{D_m k_T}{C_s C_p}\frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial \mathrm{C}}{\partial \mathrm{t}} + \mathrm{v}\frac{\partial \mathrm{C}}{\partial \mathrm{y}} = \mathrm{D_m}\frac{\partial^2 \mathrm{C}}{\partial \mathrm{y}^2} + \frac{\mathrm{D_m k_T}}{\mathrm{T_m}}\frac{\partial^2 \mathrm{T}}{\partial \mathrm{y}^2} \tag{4}$$

For the current problem, the related boundary conditions are

$$u = \mathrm{U_0(t)}, \mathrm{T} = \mathrm{T_w}, \mathrm{v} = -\mathrm{v(t)},\ C = C_w \quad \text{at} \quad \mathrm{y} = 0 \tag{5}$$

$$u = 0,\ T = \mathrm{T}_\infty, \mathrm{v} = 0,\ C = C_\infty \qquad \text{as } \mathrm{y} \rightarrow \infty \tag{6}$$

where β is the coefficient of thermal expansion, g is the gravitational acceleration, v is the component of the velocity in the y direction, β^* is the coefficient of concentration expansion, ρ is the fluid density, u is the component of the velocity along the x $-$axis, T is the fluid temperature, T_w is the wall temperature, T_∞ is the fluid temperature in the free stream, K is the permeability of the porous plate, C is the fluid concentration, $\mathrm{C_w}$ is the wall concentration, C_∞ is the free stream concentration, σ' is the electric conductivity, k is the thermal conductivity of the sheet, T_m is the mean temperature of the fluid, C_s is the concentration susceptibility, C_p is the specific heat at constant pressure, υ is the kinematic viscosity, $\mathrm{D_m}$ is the mass diffusivity coefficient, and $\mathrm{k_T}$ is the thermal diffusion ratio.

Utilizing a similarity variable σ as

$$\sigma = \sigma(t) \tag{7}$$

where equation (1) is assumed to have a solution in terms of the time-varying length scale (Hasanuzzaman et al. [8]) provided by

$$v = -v_0 \frac{\upsilon}{\sigma} \tag{8}$$

In this case, v_0 is the dimensionless normal velocity at the sheet. Here suction is indicated by $v_0 > 0$ and blowing by $v_0 < 0$, respectively.

We propose the following similarity transformation to simplify the mathematical study of the problem

$$\eta = \frac{y}{\sigma}, \quad f(\eta) = \frac{u}{U_0}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \tag{9}$$

The preceding equations (7)–(9) are applied to transform the given equations (1)–(4) into dimensionless coupled ODEs:

$$f''(\eta) + 2\xi f'(\eta) + Gr\theta(\eta) + Gm\phi(\eta) - Mf(\eta) - \frac{1}{Da} f(\eta) = 0 \tag{10}$$

$$\theta''(\eta) + Pr\left\{2\xi\,\theta'(\eta) - Ec\left(f'(\eta)\right)^2 + Df\phi''(\eta)\right\} = 0 \tag{11}$$

$$\phi''(\eta) + 2\xi Sc\phi'(\eta) + Sc\,Sr\theta''(\eta) = 0 \tag{12}$$

The converted boundary conditions are provided by:

$$f(\eta) = 1, \quad \phi(\eta) = 1, \quad \theta(\eta) = 1 \quad at\ \eta = 0 \tag{13}$$

$$f(\eta) = 0, \quad \phi(\eta) = 0, \quad \theta(\eta) = 0 \quad at\ \eta \to \infty \tag{14}$$

where Schmidt number is $Sc = \frac{\upsilon}{D_m}$, local Grashof number is $Gr = \frac{g\beta(T_w - T_\infty)\sigma^2}{U_0 \upsilon}$, Prandtl number is $Pr = \frac{\rho\upsilon C_p}{k}$, $Da = \frac{K}{\sigma^2}$ is the Darcy number, Soret number is $Sr = \frac{D_m k_T (T_w - T_\infty)}{\upsilon T_m (C_w - C_\infty)}$, Magnetic force parameter is $M = \frac{\sigma' B_0^2 \sigma^2}{\rho\upsilon}$, $Ec = \frac{1}{\rho C_p}\frac{U_0^2}{(T_W - T_\infty)}$ is the Eckert number, the Dufour number is $Df = \frac{D_m k_T (C_w - C_\infty)}{C_s C_p \upsilon (T_w - T_\infty)}$, $Gm = \frac{g\beta^*(C_w - C_\infty)\sigma^2}{U_0 \upsilon}$ is the modified local Grashof number, and $\xi = \eta + \frac{v_0}{2}$ is the time-dependent parameter.

The flow parameters are the shear stress (τ), the local Nusselt number (Nu), and the local Sherwood number (Sh) defined as: $\tau \propto f'(0)$, $Nu \propto -\theta'(0)$, $Sh \propto -\phi'(0)$ (15)

NUMERICAL SOLUTION

The main target is to utilize the Finite difference method (FDM) for solving the ODEs (10)-(12) including the boundary conditions (13)–(14) in this research. This method has been satisfied for accuracy and efficiency in solving various problems (Ali et al. [10] and Cheng and Lin [11]). The solution domain space is discretized in the FMD.

We will apply grid size $\Delta\eta = h > 0$ in η -direction, $\Delta\eta = \frac{1}{N}$, with $\eta_i = ih$ for i = 0,1,..., N. Define $f_i = f(\eta_i)$, $\theta_i = \theta(\eta_{i,})$ and $\phi_i = \phi(\eta_i)$.

At the i^{th} node, we consider F_i, $\Theta_{i,}$ and Φ_i to be the numerical values of $f, \theta,$ and ϕ, respectively. Hence, we suppose:

$$f'|_i = \frac{f_{i+1} - f_{i-1}}{2h}, \quad \theta'|_i = \frac{\theta_{i+1} - \theta_{i-1}}{2h}, \quad \phi'|_i = \frac{\phi_{i+1} - \phi_{i-1}}{2h} \tag{16}$$

$$f''|_i = \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2}, \quad \theta''|_i = \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{h^2}, \quad \phi''|_i = \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{h^2} \tag{17}$$

By applying FDM, the system of ODES (10)-(12) is discretized in space which is called the main step. To do this we put (16) - (17) into (10)-(12) and neglect the truncation errors. So for (*i = 0, 1,..., N*), the resulting algebraic equations take the form:

$$F_{i+1} - 2F_i + F_{i-1} + \xi h(F_{i+1} - F_{i-1}) + Grh^2\Theta_i + Gmh^2\Phi_i - Mh^2F_i - \frac{h^2}{Da}F_i = 0 \tag{18}$$

$$\Theta_{i+1} - 2\Theta_i + \Theta_{i-1} + Pr\left[\xi h(\Theta_{i+1} - \Theta_{i-1}) + D_f(\Phi_{i+1} - 2\Phi_i + \Phi_{i-1}) - Ec(F_{i+1} - F_{i-1})^2\right] = 0 \tag{19}$$

$$\Phi_{i+1} - 2\Phi_i + \Phi_{i-1} + Sc[\xi h(\Phi_{i+1} - \Phi_{i-1}) + Sr(\Theta_{i+1} - 2\Theta_i + \Theta_{i-1})] = 0 \tag{20}$$

Also, the boundary conditions are

$$F_0 = 1,\ \Theta_0 = 1,\ \Phi_0 = 1,\ F_N = 0,\ \Theta_N = 0,\ \Phi_N = 0 \tag{21}$$

The equations from (18) to (20) represent a nonlinear system of algebraic equations in F_i, Θ_i, and Φ_i. We will apply the Newton iteration

RESULTS AND DISCUSSIONS

The roles of variable permeability on the time-dependent MHD free convective transport upon a vertical perforated plate with viscous dissipation have been analyzed numerically in this study. The ODEs (10)-(12) are solved numerically by using the FDM. The dimensionless fluid temperature, velocity, and concentration fields for separate values of the dimensionless parameters or numbers are revealed in Figures 2-9**.** The numerical values like the local skin friction coefficient $\left(f'(0)\right)$, the heat transfer rate $(-\theta'(0))$, and the mass transfer rate $\left(-\phi'(0)\right)$ are given in Tables 1-3. We considered $Ec = 0.5, M = 0.5,\ Pr = 0.71,\ Da = 0.5, Gr = Gm = 10.0, Df = 0.5, Sr = 2.0$, and $Sc = 0.22$ in the whole simulation.

With an enhancement in the Darcy number (Da), the fluid velocity improves as shown in Figure 2. The Darcy number measures the permeability of the plate. As the permeability of the plate improves the values of Da enhance. Therefore, the fluid motion goes up for growing values of Da. This is because the fluid gets more space to flow for the large permeability of the plate. As a result, the fluid velocity exacerbates. The fluid motion is enhanced due to the increasing the local modified Grashof number (Gm) as shown in Figure 3. The positive value of Gm>0 shows the system is heated. It means that increasing values of Gm in the heating plate improves the fluid velocity. The momentum boundary layer thickness diminishes for growing values of Gm. The cooling system is observed for the negative value of Gm<0. It means that the cooling plate decays the fluid motion for growing values of Gm. The symmetrical shape is found for the combined values of (Gm>0) and (Gm<0).

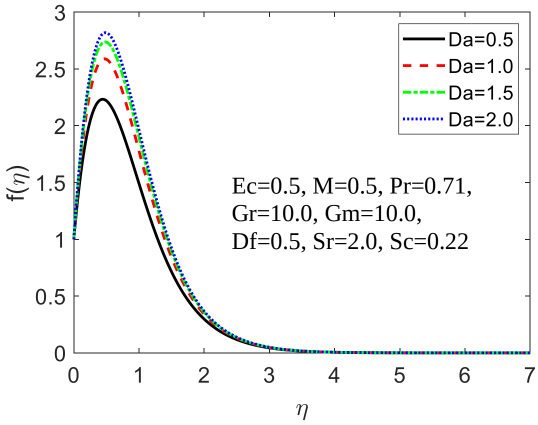

FIGURE 2. Velocity profile for Da

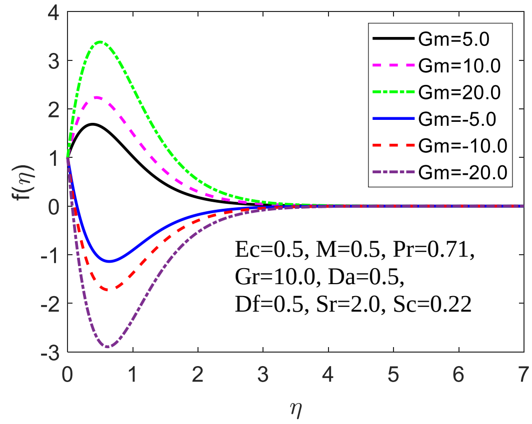

FIGURE 3. Velocity profile for Gm

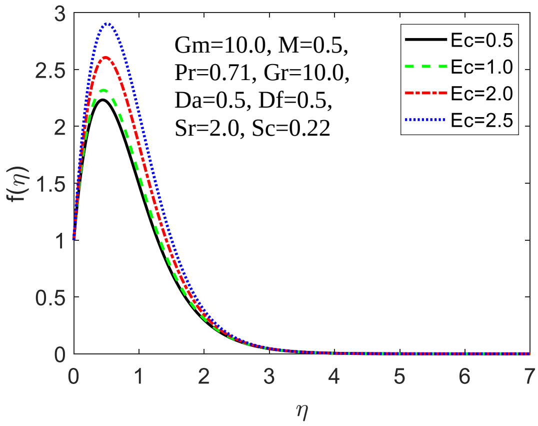

FIGURE 4. Velocity profile for Ec

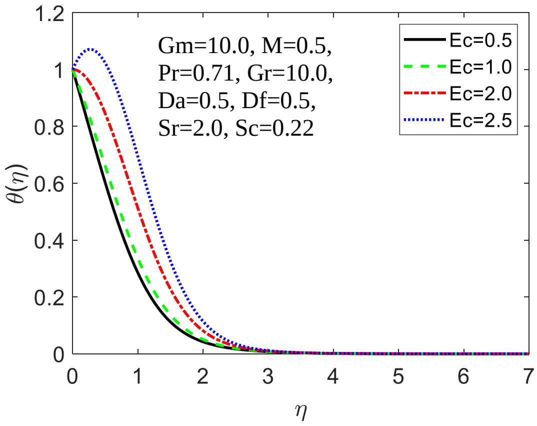

FIGURE 5. Temperature profile for Ec

It is evident from Figure 4 that f(η) goes up for improving values of the Eckert number (Ec). This is because by including the viscous dissipation influence (Ec), mechanical energy is transferred into thermal energy. So, the fluid becomes thinner and consequently, its velocity is improved. The fluid temperature within the boundary layer improves with the enhancement in the Eckert number as shown in Figure 5. The Eckert number (Ec) is a significant part of studying the thermal behavior of fluid flow. As a result, the thermal boundary layer becomes thicker and lessens heat dissipation. This is because by including the viscous dissipation effect of Ec, the mechanical energy is changed into thermal energy. This thermal energy improves the temperature field. The fluid velocity goes down with the improvement of the magnetic field parameter (M) as shown in Figure 6. This magnetic force field tends to impede the convective fluid velocity. Hence, the imposition of a magnetic field on the flow field creates a Lorentz force which in turn reduces the fluid velocity away from the sheet surface. Consequently, the values of f′0 lessen at the wall for rising values of M. This result demonstrates that the magnetic field can be utilized to regulate the flow. The fluid motion goes down for growing values of the Prandtl number (Pr) as shown in Figure 7. We know that the mathematical relation is $Pr = \frac{\rho \upsilon C_p}{k}$. The Prandtl number directly varies with the kinematic viscosity. The viscous forces tend to improve the buoyancy forces as Pr goes up. These forces produce slow velocity in the boundary layer. That is why, fluid movement is not so easy in the computational area. Hence the fluid motion is observed to lessen and the boundary layer thickness is found to decay for large values of Pr. The Prandtl number (Pr) inversely varies with the thermal diffusion. With an enhancement in Pr, the thermal diffusion lessens. So, the thermal boundary layer becomes thinner. For this reason, the fluid temperature reduces for growing values of Pr as demonstrated in Figure 8. Physically, the fluid with a larger Pr gives a higher heat capacity.

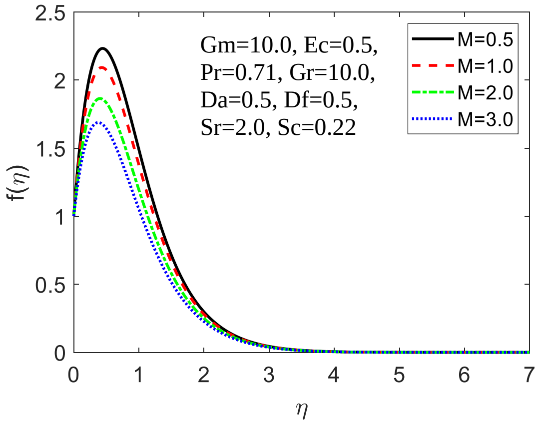

FIGURE 6. Velocity profile for M

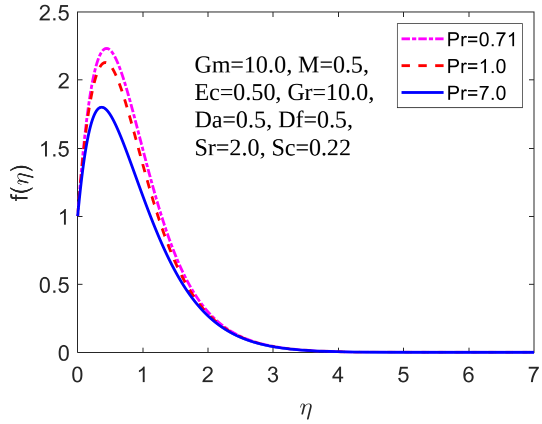

FIGURE 7. Velocity profile for Pr

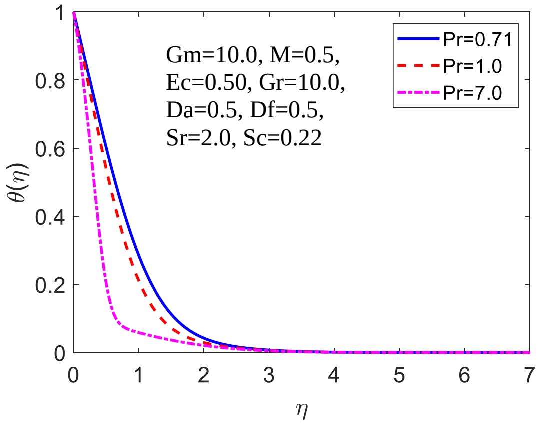

FIGURE 8. Temperature profile for Pr

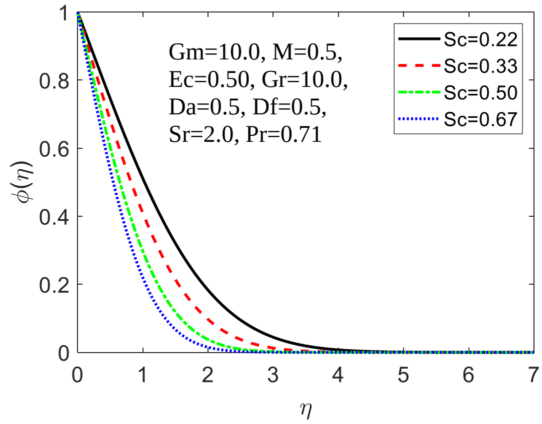

FIGURE 9. Concentration profile for Sc

The role of the separate values of Schmidt number (Sc) on $\phi(\eta)$ is decorated in Figure 9. The Schmidt number inversely varies as the molecular (species) diffusivity. It is noticed from Figure 9 that the concentration field goes down for growing values of Sc. The associated reduction in mass diffusivity provides a small forceful mass transfer that reduces the density gradient. So, the density boundary layer thickness reduces for Sc. This is the reason the mass transfer uses the interplay with the concentration distribution, and the velocity profile of the material can be dominated via Sc.

TABLE 1. Influence of various values of the Eckert number (Ec) on $-\phi'(0)$, $f'(0)$and $-\theta'(0)$

Ec	$-\theta'(0)$	$f'(0)$	$-\phi'(0)$
0.5	0.950135283647402	6.26357787458863	0.507185477222976
1.0	0.813178797235991	6.43009257573554	0.507185477222976
2.0	0.612041125208695	6.75412094622520	0.507185477222976
2.5	0.537921752273820	6.90953175361174	0.507185477222976

Table 1 depicts the influence of various values of the Eckert number (Ec) on $-\theta'(0)$, $f'(0)$ and $-\phi'(0)$. The values of $f'(0)$ improve but the values of $-\theta'(0)$ decay for growing values of Ec. The increases by about 10% and the values of $-\theta'(0)$ lessen by about 43% due to increasing values of Ec from 0.5 to 2.5. For this reason, the fluid motion and temperature accelerate owing to increased values of Ec.

COMPARABLE TABLES

The numerical result of the existing research has been compared with previously published work which is included in Table 3. Numerical outcomes for the amounts of the heat transmission rate at the plate are compared with the findings of Hasanuzzaman et al. [8] for the case of base fluid to confirm the validity and correctness of the result obtained.

TABLE 3. Values of $-\phi'(0)$ and $-\theta'(0)$ for v_0 and Df

v_0	Df	$-\phi'(0)$ Hasanuzzaman et al. [8]	$-\varphi'(0)$ Present study	$-\theta'(0)$ Hasanuzzaman et al. [8]	$-\theta'(0)$ Present study
0.5	0.2	0.22187363	0.219355	1.41983	1.49929
0.5	0.5	0.13702065	0.133550	1.48232	1.36665

CONCLUSIONS

We have analyzed the important roles of the Eckert number and the Darcy number on hydro-magnetic convective heat and mass transport upon a vertical perforated plate. The following remarks can be drawn:

- With growing values of M, the velocity of the fluid particle diminishes.
- The fluid motion and the temperature lessen for uplifting amounts of Pr.
- With an increment in Sc, the fluid concentration diminishes.
- The fluid motion goes up quickly for moving values of Da.
- With growing values of the Eckert number, the fluid motion and temperature go up quickly.
- The values of the local skin friction coefficient increase by about 10% but the heat transfer rate lessens by about 43% due to rising values of the Eckert number from 0.5 to 2.5.

REFERENCES

1. K. B. Pavlov, “Magnetohydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface”, (Magnitnaya Gidrodinamika, 4(1), 1974), 146-147.
2. Y. S.Taiwo, and G.Dauda, “Impact of heat generation/absorption on transient natural convective flow in an annulus filled with porous material subject to isothermal and adiabatic boundaries”, (GEM-International Journal on Geomathematics, 10, 2019), 1-16.
3. L. Prandtl, “Über Flussigkeitsbewegung bei sehr kleiner Reibung”, (Verhandl. III, Internat. Math.-Kong., Heidelberg, Teubner, Leipzig, 1904, 1904), 484-491.
4. A. J. Fonsho, “The effect of Dissipation function and variable fluid Properties on unsteady MHD flow of a radiating gas inside a vertical thermal (Annual conference of MAN at Usmanu Danfodiyo University Sokoto, 3rd September, 2004).
5. R.Sen, “On Visco-Elastic free convection Boundary Layer flows past an infinite plate with constant suction”, (Indian journal of pure and applied Mathematics, 9(3), 1977), 229-241.
6. I. J. Uwanta, B. Y. Isah, and M. O. Ibrahim, “Viscoelastic fluid flow past an infinite vertical plate with heat dissipation”, (International Journal of Computer Applications, 36(2), 2011), 17-24.
7. A. O. Ajibade, and A. M. Umar, “Effects of viscous dissipation and wall conduction on steady mixed convection Couette flow of heat generating/absorbing fluid”, (International Journal of Applied Mechanics and Engineering, 24(4), 2019),12-35.

8. M. Hasanuzzaman, M. Azad, A. Kalam, and M. Hossain, “Effects of Dufour and thermal diffusion on unsteady MHD free convection and mass transfer flow through an infinite vertical permeable sheet”, (SN Applied Sciences, 3(12), 2021), pp. 1-11.
9. M. Hasanuzzaman, S. Akter, S. Sharin, M. M. Hossain, A. Miyara, and M. A. Hossain, “Viscous dissipation effect on unsteady magneto-convective heat-mass transport passing in a vertical porous plate with thermal radiation”, (Heliyon, 9, 2023), e14207.
10. J. C. Ali, E. A. Sameh, and S. A. Abdulkareem, “Melting and radiation effects on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing eternal flows”, (International Journal of Physical Sciences, 5(7), 2010),1212-1224.
11. W. T. Cheng, and C. H. Lin, “Unsteady mass transfer in mixed convective heat flow from a vertical plate embedded in a liquid-saturated porous medium with melting effect”, (International Communications in Heat and Mass Transfer), 35(10), 2008),1350-1354.