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Permeability Impact on Unsteady Magneto-convective Heat-mass Transference by Micropolar Binary Mixture Passing an Inclined Plate

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Abstract. This research paper has studied the impacts of varying permeability on time-dependent MHD convective transport across an inclined porous plate. We have used the local similarity transformation to convert the governing partial differential into ordinary equations. Following that, using the shooting technique and the software "MATLAB ODE45", a numerical solution has been found for the dimensionless ordinary differential equations with boundary conditions. The impacts of relevant parameters or numbers such as Darcy number (*Da*), Prandtl number (*Pr*), thermal Grashof number (*Gr*), and Schmidt number (*Sc*) on the velocity, temperature, concentration, and microrotation of the fluid have been studied within the boundary layer. The growing Darcy number (*Da*) increases fluid velocity, whereas the microrotation field exhibits the opposite behavior. The liquid rate and temperature reduce with the variation of the Prandtl number (*Pr*). With Da (0.5-3.0), the surface couple stress drops by around 28% while the skin friction coefficient improves by around 17%.

Keywords: Permeability, MHD, Micropolar binary mixture, Heat and mass transfer.

INTRODUCTION

Permeability is an essential component in fluid flow phenomena. The ability of the formation to transport fluids is measured by its permeability, an inherent characteristic of the porous medium. The applications of fluid movement over porous media occur significantly in the recovery of oil, gas, and minerals from Mother Earth, geophysical flow, transport, and sequestration of contaminants in the subsurface. It is also beneficial for large-scale chemical reactions, including catalysts, filters, adsorbents, and models of physiological processes. The effects of joule heating and thermal diffusion on the flow of a micropolar fluid through an infinitely porous vertical medium during an unsteady hydromagnetic unrestrained convective heat and mass transmission were examined by Haque et al. [1]. The unstable hydromagnetic unconstrained convection circulation of a fluid via a continuous plate in a permeable medium with variable heat and mass transport was exactly solved by Ali et al. [2]. Using a micropolar fluid moving across a continuous porous surface, Hossain et al. [3] investigated the properties of the inclined angle and Lorentz force on unstable convective thermal-material transmission. Magnetohydrodynamics is shortly expressed as MHD. MHD is the learning of magnetic actions and behavior of fluids that are electrically conducting. Plasma, Liquid metals (such as mercury), electrolytes, and seawater are examples of these magnetic fluids. Its applications are extensive and span a wide range of technical domains, including nuclear reactors for geothermal energy extraction, plasma studies, MHD generators and pumps, boundary layer flow control, and boundary layer flow control. Rao et al. [4] directed an analysis on the properties of chemical processes on an unstable magnetic hydrodynamics convection flow of fluid across a vertical plate submerged in a permeable material with heat absorbing. The consequences of thermal radiation and the rotation on an infinitely moving absorbent plate's turbulent MHD flow were studied in experiments by Krishna et al. [5]. The impacts of radiative and viscous dissipation on the movement of a magnetized conductive heat-mass transmission through a vertical permeable sheet were investigated by Hasanuzzaman et al. [6].

A binary mixture combines two liquids that wouldn't normally mix. Simple vinaigrette), oil and water, and Hollandaise sauce are all excellent examples of binary fluid mixtures. These binary fluid mixtures show a vital role in industrial production. The impact of unstable convection with radiative heat transmission, chemical reaction, and thermophoresis in a micropolar fluid past a vertical permeable plate over a binary mixture was investigated by Animasaun [7]. In two dimensions of MHD boundary layer movement on a vertical surface, Sharma and Nath [8] examined the influences of thermal diffusion, heat source, chemical reaction, and viscous dissipation. The transfer of energy takes place through the transmission of heat and mass. Conduction, convection, and radiation are the methods through which heat is moved as a result of temperature differences. Due to differences in density and pressure, mass can be transmitted through absorption, adsorption, and stream. Rahman and Salahuddin [9] performed a computational investigation of the impact of varying temperature depending viscosity and electric conductivity over a continuous 2D convective movement of an incompressible fluid on an inclined permeable sheet. Hossain et al. [10] looked into the implications of thermal radiation on unstable magnetic and convective heat - mass transmission through micropolar binary mixing of fluid across an inclined permeable surface.

This investigation mainly aims to study the impact of varying permeability on time-dependent MHD convective transport along an inclined porous plate. The key novelty of this research work is to consider an inclined permeable plate with the impact of variable permeability. In MATLAB software, the shooting method is used to visually display the outcomes of the numerical solutions for the dimensionless equations like velocity, micro-rotation, temperature, and concentration equations. The rates of mass and heat transmissions along with the local skin friction coefficient are also provided in the tabular representations.

MATHEMATICAL MODELING

The impacts of variable permeability on time-dependent MHD convective transportation over an inclined permeable plate were studied in this paper. The fluid flow is considered across the *x*-axis, which is taken along a continuous permeable surface, and *y*-axis is normal to it. The inclination of permeable surface from horizontal is assumed to be an angle (γ), which will determine three positions (horizontal, vertical and inclined surfaces) of the permeable surface at a time to compare the impacts of several parameters or numbers. The velocity components are assumed to be the function of y and t. The suction $v(t)$ is considered the outward direction from the permeable plate. A magnetic field with uniform strength B_0 will be applied transversely to the direction of the flow. The dimensional surface temperature (T_w) will be considered less than the temperature at the free stream (T_∞). Also, the dimensional surface concentration (C_w) will be supposed to be less than the concentration at the free stream (C_∞).

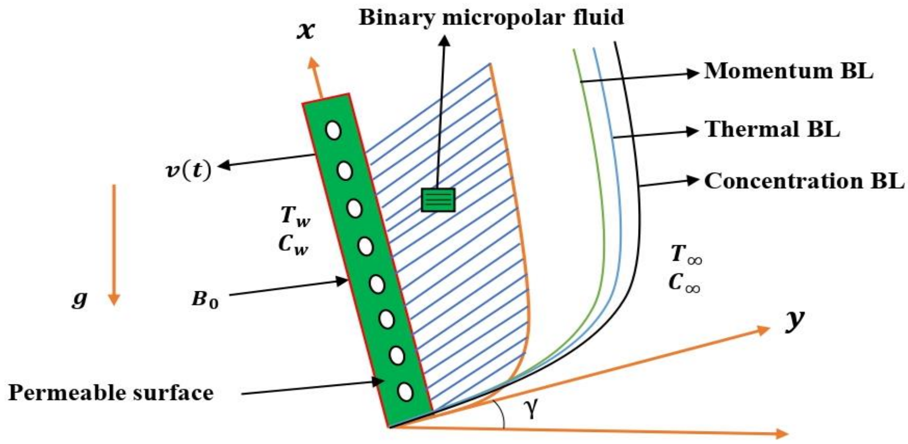


FIGURE 1. Coordinate system along with physical model

We have the following governing equations with all suppositions stated above and Boussinesq's approximation:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \left(\frac{\mu + \tau}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\tau}{\rho}\frac{\partial N}{\partial y} + g\beta(T - T_\infty)cos\gamma + g\beta^*(C - C_\infty)cos\gamma - \frac{\sigma {B_0}^2}{\rho}u - \frac{\nu}{K}u \tag{2}$$

In this case, we have imposed a uniform transverse magnetic field B_0 which is normal to the porous surface.

$$\rho C_p \left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right) = \kappa \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} + \frac{\partial}{\partial y}[V_T(C - C_\infty)] = D_m \frac{\partial^2 C}{\partial y^2} \tag{4}$$

$$\frac{\partial N}{\partial t} + v\frac{\partial N}{\partial y} = \frac{\gamma^*}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{\tau}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \tag{5}$$

Equations (2) – (5) are subjected to the following boundary conditions:

$$u(y,0) = 0 \quad T(y,0) = T_w \quad N(y,0) = 0 \quad C(y,0) = C_w \quad \text{for } t \le 0$$

$$u(0,t) = 0 \quad T(0,t) = T_w \quad N(0,t) = m_0 \frac{\partial u}{\partial t} \quad C(0,t) = C_w \quad \text{for } t > 0$$

$$u(\infty,t) \to U_0 \quad T(\infty,t) \to T_\infty \quad N(\infty,t) \to 0 \quad C(\infty,t) \to C_\infty \quad \text{for } t > 0$$

The velocity component can be stated as follows-

$$v = -c\left(\frac{\nu}{t}\right)^{1/2} \tag{6}$$

where $c < 0$ and $c > 0$ denote the injection and suction parameter, respectively.

The following relations show the spin gradient viscosity and micro-inertia per unit mass:

$$j = \frac{\mu}{\rho U_0} \quad \text{and} \quad \gamma^* = \left(\mu + \frac{\tau}{2}\right) j \tag{7}$$

We can also define the thermophoretic parameter as-

$$V_T = -\frac{K^{Th}}{T_{ref}} \cdot \frac{\partial T}{\partial y} \tag{8}$$

Following Boussinesq [11], we can write the buoyancy model as-

$$-\frac{\partial p}{\partial x} = g\beta\rho_\infty(T_\infty - T) + g\beta^*\rho_\infty(C_\infty - C) \tag{9}$$

The following dimensionless variables can be introduced into (2) - (5):

$$\eta = \frac{y}{2\sqrt{\nu t}}; \quad f(\eta) = \frac{u}{U_0}; \quad \theta(\eta) = \frac{T}{T_\infty}; \quad \varphi(\eta) = \frac{C}{C_\infty}; \quad h(\eta) = \frac{N\sqrt{\nu t}}{U_0} \tag{10}$$

With the assistance of Animasaun and Anselm [12], we have finally the following dimensionless non-linear ODEs:

$$(1 + \xi - \theta\xi + k_1)f'' - \xi\,\theta' f' + 2(\eta + c)f' + 2k_1 h' + G_r\xi(1 - \theta)cos\gamma + G_c\xi(1 - \varphi)cos\gamma - Mf - \frac{1}{Da}f = 0 \tag{11}$$

$$\theta'' + 2Pr(\eta + c)\theta' = 0 \tag{12}$$

$$\varphi'' + 2S_c(\eta + c)\varphi' - \lambda S_c(1 - \varphi)\theta'' + \lambda S_c\theta'\varphi' = 0 \tag{13}$$

$$\left(1 + \xi - \theta\xi + \frac{k_1}{2}\right)h'' + 2(\eta + c)h' + 2h - \frac{8L_1}{1 + \xi - \theta\xi}h - \frac{2L_1}{1 + \xi - \theta\xi}f' = 0 \tag{14}$$

The dimensionless associated boundary conditions are obtained as:

$$f(\eta) = 0, \quad \theta(\eta) = \theta_w, \quad \varphi(\eta) = \varphi_w, \quad h(\eta) = -\frac{1}{4}f'(0) \quad at\ \eta = 0 \tag{15}$$

$$f(\eta) \to 1, \quad \theta(\eta) \to 1, \quad \varphi(\eta) \to 1, \quad h(\eta) \to 0 \quad at\ \eta \to \infty \tag{16}$$

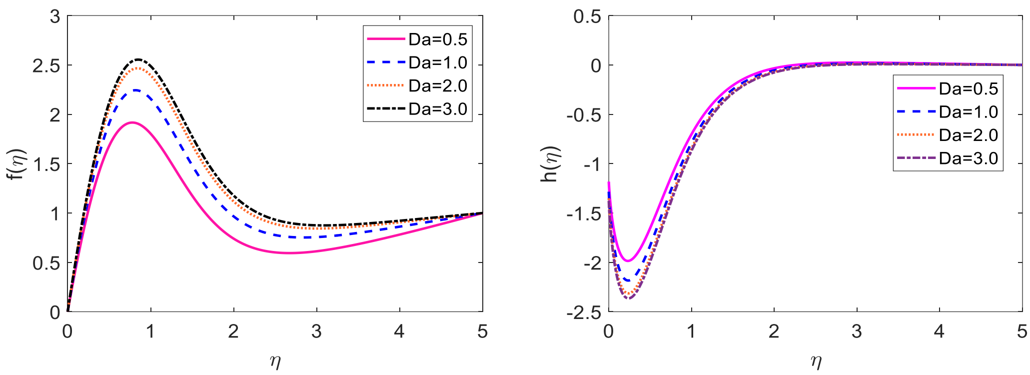
where $\theta(\eta)$, $\varphi(\eta)$, $f(\eta)$, and $h(\eta)$ denote the dimensionless temperature, concentration, velocity, and micro-rotation functions, respectively. Also, η is the similarity variable, $k_1 = \frac{\tau}{\mu}$ is the micro-rotation parameter, γ is the inclined angle of permeable surface with y-axis, $\xi = bT_\infty$ is the variable viscosity parameter, $Gr = \frac{4tg\beta}{U_0 b}$ is the thermal Grashof number, $\lambda = -\frac{\kappa^{Th}T_\infty}{\nu T_{ref}}$ is the thermophoretic parameter, $Gc = \frac{4tg\beta^*}{U_0 b}$ is known as the solutal Grashof number, $M = \frac{4\sigma B_0^2 L_1}{\rho k_1 U_0}$ is the magnetic field parameter, $Pr = \frac{\nu}{\gamma} = \frac{\mu C_p}{\kappa}$ is the Prandtl number, $L_1 = k_1 U_0 t$ is defined as the time dependent micro-rotation parameter, $Da = \frac{K k_1 U_0}{4\nu L_1}$ is the Darcy number, and $Sc = \frac{\nu}{D_m}$ is the Schmidt number.

RESULTS AND DISCUSSION

An investigation of variable permeability effect on time-dependent magnetic and convective heat-mass transmission by micropolar binary mixture over an inclined permeable plate has been familiarized in this research work. The first step was to convert the higher order non-linear PDEs (1) through (5) into 2nd order coupled non-linear ODEs. The initial value problem has also been created by using the similarity method. The associated nonlinear ODEs (11) through (14) with boundary conditions (15) through (16) have now been numerically resolved. Here the shooting technique has been applied with "ODE45 MATLAB" software to obtain desired numerical outcomes. The graphs 2 to 4 present the numerical outcomes of the current study. The parameters or numbers like Darcy number (*Da*), thermal Grashof number (*Gr*), thermophoretic parameter ($\boldsymbol{\lambda}$), Schmidt number (*Sc*), temperature dependent variable viscosity parameter (ξ), suction parameter (c), solutal Grashof number (*Gc*), Prandtl number (*Pr*), inclined angle (γ), micro-rotation parameter (K_1) time dependent micro-rotation parameter (L_1), and magnetic parameter (M) have been included in this study. The amounts for the parameters or numbers have been fixed as: $Da = 1.0$, $\gamma = 45^0$, $M = 2.0$, $Pr = 0.71$, $\boldsymbol{\lambda} = 1.00$, $Sc = 0.22$, $L_1 = 0.33$, $K_1 = 0.5$, $\xi = 3.00$, $Gr = 10.0$, $c = 0.5$, $Gc = 10.0$. Also, a finite point $\eta = 5.0$ has been chosen for the boundary conditions at infinity.

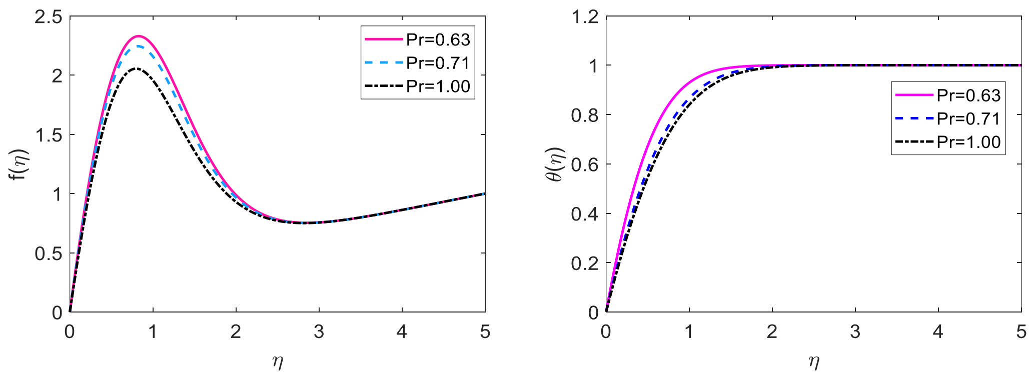
The contributions of Darcy number (*Da*) on the fluid's velocity, and micro-rotation fields are depicted in Fig. 2(a-b). The Darcy number is clearly proportional to the permeability of the plate based on the mathematical relationship. The porosity of the plate improves as the Darcy number rises. It indicates that the porosity of the plate has little effect on the flow. Because of this, Fig. 2(a) shows that elevated amounts of Darcy number result in an upsurge in fluid velocity. However, the microrotation distribution exhibits an opposite outline in Fig. 2(b). This is as a consequence of the existence of porous material, which raises flow resistance and results a lessening in fluid flow.

The profiles of velocity and temperature are influenced by the Prandtl number (*Pr*), as presented in Fig. 3(a-b). The Prandtl number is measured by the viscosity to thermal conductivity ratio. As illustrated in Fig. 3(a), the fluid's viscosity improves as the Prandtl number rises. As a result, the fluid particles cannot move freely. The outcome is a reduction in fluid velocity. Also, the Prandtl number (*Pr*) and thermal conductivity are inversely associated. Following the temperature distribution in Fig. 3(b), it appears that the temperature declines as the value of *Pr* grows. In physical meaning, advanced amounts of Prandtl number diminish the thermal conductivity, which lessens heat conduction and finally decreases the temperature.



(a) (b)

FIGURE 2. Profiles of (a) Velocity (b) Micro-rotation for several amounts of Darcy number (*Da*)

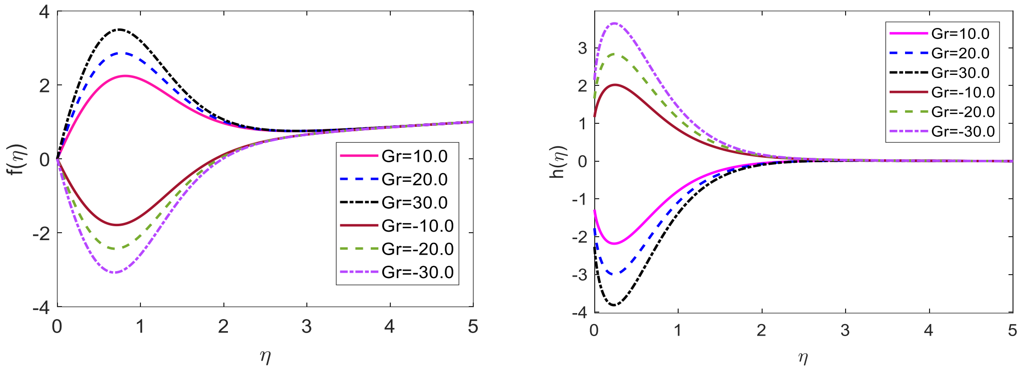


(a) (b)

FIGURE 3. Profiles of (a) Velocity (b) Temperature for several amounts of Prandtl number (*Pr*)

Fig. 4(a-b) describes the velocity and micro-rotation fields for numerous amounts of thermal Grashof number (*Gr*). As the amount of *Gr* improves, Fig. 5(a) displays how the fluid accelerates. The ratio of the buoyant force imposed on the fluid to the viscous force in the boundary layer is measured by the thermal Grashof number. As a result, an escalation in *Gr* leads to lessen the drag force and hence the velocity fields upgrade in the boundary layer. A symmetrical structure is produced from the velocity field for both positive and negative amounts of *Gr*. The system is heating for $Gr < 0$ and cooling for $Gr > 0$. Fig. 5(b) shows how the thermal Grashof number affects the distributions of microrotations. The microrotation fields of the fluids in the vicinity of the wall ($0.0 \le \eta \le 2.2$)

reduce from the negative side as they climb from zero to positive values. Following that, *Gr* has no effect on the microrotation fields, and then asymptotically tends to 0 as $\eta \rightarrow 5$. As a result, fluids present in the domain have a significant impact on the micro-rotation patterns.



(a) (b)

FIGURE 4. Profiles of (a) Velocity (b) Micro-rotation for several amounts of thermal Grashof number (*Gr*)

Skin friction, Nusselt number, Sherwood number and surface couple stress

The thermophysical quantities as the Nusselt number (*Nu*), skin friction coefficient (*Cf*), surface couple stress (*Cs*), and Sherwood number (*Sh*) of practical importance which correspond to $\theta'(0)$, $f'(0)$, $h'(0)$, and $\varphi'(0)$ respectively are displayed in tabular forms to elucidate the fluid's inner properties as shown Table 1.

TABLE 1. Numerical values of thermophysical quantities for several amounts of Darcy number (*Da*)

Da	$f'(0)$	$-\theta'(0)$	$-\varphi'(0)$	$h'(0)$
0.5	4.60279765585275	1.29973372945962	0.824898149336346	-4.00966136707747
1.0	5.01349283836710	1.29973372945854	0.824898149490593	-4.46452320581181
2.0	5.28183125108293	1.29973372945842	0.824898149534336	-4.76468384612630
3.0	5.38428141698421	1.29973372945826	0.824898149492958	-4.87979491732699

Table 1 demonstrates the impacts of Darcy number (*Da*) on *Nu*, *Cf*, *Cs*, *Sh*. It is obvious from Table 1 that *Nu* improves and *Cs* declines with the growing amounts of $Da(0.5 - 3.0)$. But a few changes are noticed in heat and mass transfer rates with variation of $Da(0.5 - 3.0)$. We further observe that *Nu* is advanced by around 17% but *Cs* is diminished by around 28% for growing amounts of $Da(0.5 - 3.0)$.

Comparison

The numerical outcomes of this study have been compared to Animasaun [7]. Table 2 compares the mass transmission rates for the Schmidt number (*Sc*) between the published article and the current numerical consequences. The comparison shows a good agreement with Animasaun [7].

TABLE 2. For Schmidt number (*Sc*), showing the comparison of local mass transmission rate $-\varphi'(0)$

Sc	*c*	$-\varphi'(0)$ [Present study]	$-\varphi'(0)$[Animasaun [7]]	Error
0.22	0.0	0.590842078810744	0.582727235867182	1.00 %
0.42	0.0	0.870115821679177	0.860039307005880	1.00 %
0.62	0.0	1.109770363259520	1.099421579293592	1.00 %
0.22	0.5	0.797248614856556	0.798091264284715	0.10 %
0.42	0.5	1.272764843684690	1.279330713448670	0.50 %
0.62	0.5	1.714208941493380	1.728144872634346	0.75 %

CONCLUSIONS

- The elevated amounts of Darcy number (*Da*) result in an upsurge in fluid velocity, where the microrotation distribution exhibits an opposite outline.
- The temperature and fluid velocity both decline with the variation of Prandtl number (*Pr*).
- The symmetrical structures are produced from the velocity and micro-rotation fields for both positive and negative amounts of thermal Grashof number (*Gr*).

- The surface couple stress drops by around 28% while the skin-friction coefficient improves by around 17% in advance of Da $(0.5 - 3.0)$.

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